Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name:Transform Theory

Subject Code: 4SC05TTE1 **Branch: B. Sc. (Mathematics)**

Semester: 5 **Date: 11/12/2015** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Evaluate $\int_0^\infty e^{-3t}\ t\ dt$.

b) Check whether given function is even or odd?

$$f(x) = \begin{cases} 0; & -2 < x < -1 \\ k; & -1 < x < 1 \\ 0; & 1 < x < 2 \end{cases}$$

c) State First shifting theorem.

d) In the Fourier series expansion of $f(x) = |x|$ in $(-\pi, \pi)$, the value of b_n is ……

e) $L[\,f'(t)] =$ …….. .

f) If $F(\lambda)$ is the Fourier transform of $f(t)$, $F[f(x-a)] =$…….

g) $L^{-1}\{\int_s^\infty \bar{f}\,(u)\ du\} =$…… .

h) Define: Periodic function.

i) Write formula of $Z(a^n)$.

j) Write formula of Inverse Fourier transform.

k) Finite Fourier cosine transform of $f(x) = 1$ in $(0, \pi)$ is zero. Determine whether statement is True or False?

l) Z-transform of unit impulse sequence is $\frac{z}{z-1}$. Determine whether statement is True or False?

m) If $f(t)$ is a periodic function with period T, then $L[f(t)] = \int_0^T e^{-st} \cdot f(t)\, dt$. Determine whether statement is True or False?

n) Z-transform is linear. Determine whether statement is True or False?

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and prove Convolution theorem. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s-4)(s+3)}\right\}$. **(07)**

b) Find the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. **(07)**

Q-3 **Attempt all questions** **(14)**

a) State and prove Euler's formulae for Fourier series expansion of a function $f(x)$. **(07)**

b) Find the Fourier cosine transform of $(x) = \frac{1}{1+x^2}$. **(07)**

Q-4 **Attempt all questions** **(14)**

a) Find the Laplace transform of $f(t) = t^3 + e^{-3t} + t^{\frac{3}{2}} + 3^t$. **(05)**

b) Find the Fourier series of the function $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$. Also deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$. **(05)**

c) Prove that $Z[\cos n\theta] = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$ and $[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$. **(04)**

Q-5 **Attempt all questions** **(14)**

a) Find the Laplace transform of $f(t) = \frac{\cos at - \cos bt}{t} + t\sin at$. **(05)**

b) If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, then evaluate u_2 and u_3. **(05)**

c) Find the finite Fourier cosine transform of $f(x) = 2x$, $0 \le x \le 4$. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Find the Laplace transform of **(05)**

$$i)\ f(t) = \begin{cases} 0; & 0 \le t < k \\ 1; & t \ge k \end{cases}, \quad ii)\ f(t) = [t]$$

b) Find sine and cosine integral of $f(x) = e^{-kx}$. **(05)**

c) Find $Z\left[{}^{n+p}C_p\right]$, $(0 \le p \le n)$. **(04)**

Q-7 **Attempt all questions** **(14)**

a) Prove that if $Z[u_n] = U(z)$, then $Z[u_{n-k}] = z^{-k}\ U(z)(k > 0)$. **(05)**

b) Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+3)}$. **(05)**

c) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. **(04)**

Q-8 **Attempt all questions** **(14)**

a) By using the method of Laplace transform, solve the initial value problem **(05)**
$y'' + 2y' + y = e^{-t}; y(0) = -1 \text{and} y'(0) = 1.$

b) Prove that $F_s\left(x\, f(x)\right) = -\frac{d}{d\lambda}[F_c(\lambda)]$and $F_c\left(x\, f(x)\right) = \frac{d}{d\lambda}[F_s(\lambda)]$. **(05)**

c) Find $Z\left[3n - 4\sin\frac{n\pi}{4} + 5a\right]$. **(04)**